

# Time Series Based Semi-Analytical Solution of Power Systems and its Application in Direct Methods

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### Introduction

- A semi-analytical solution to power system differential equations.
- The multi-machine power system is decoupled into a set of independent single-machine-infinite-bus (SMIB) systems.
- Solving set of linear equations provides a semi-analytical solution to power system.
- The semi-analytical solution can replace Time Domain Simulation to determine the fault-on trajectory used in Direct Method

# Decoupling

#### **Decouplability Assumption**

 A multi-machine power system can be always decoupled into a set of independent SMIB systems.

#### System model

- Constant impedance model for all loads
- Classic model for all generators in the System

#### **Principle**

$$\ddot{\beta}_{1} = \frac{\alpha_{0}}{2H_{1}} \left( P_{M,1} - E_{1}^{2}G_{1} - \sum_{j=1, j\neq 1}^{m} C_{1j} \sin \delta_{j} + D_{1j} \cos \delta_{j} \right)$$

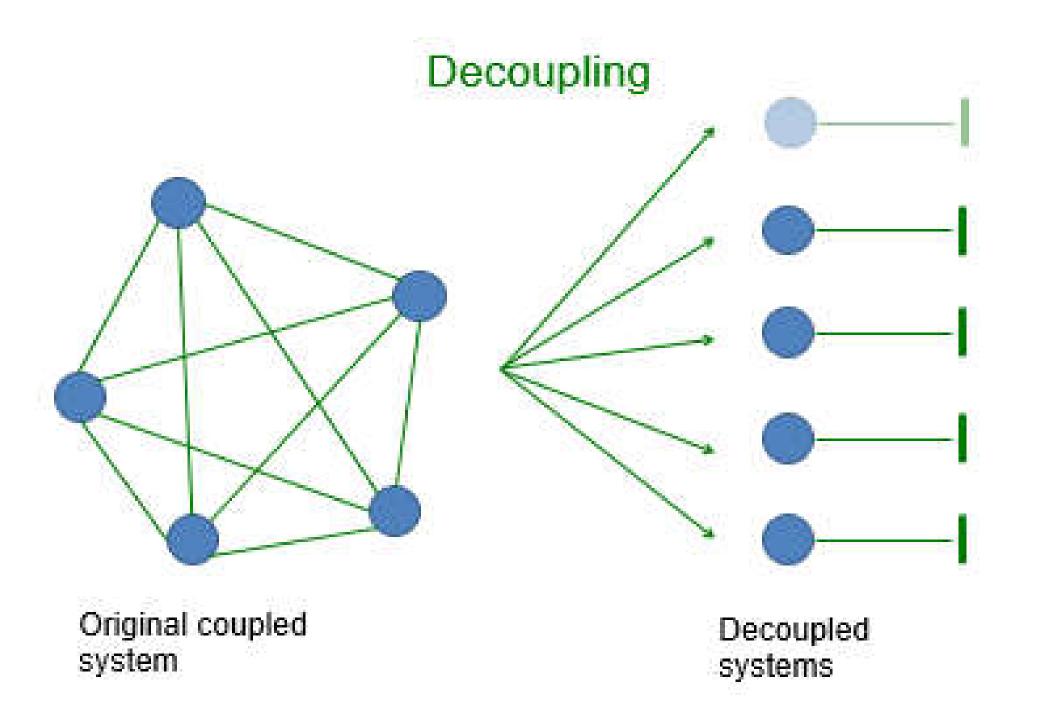
$$\ddot{\beta}_{2} = \frac{\alpha_{0}}{2H_{2}} \left( P_{M,2} - E_{2}^{2}G_{2} - \sum_{j=1, j\neq 2}^{m} C_{2j} \sin \delta_{2j} + D_{2j} \cos \delta_{2j} \right)$$

$$\ddot{\beta}_{m} = \frac{\alpha_{0}}{2H_{m}} \left( P_{M,m} - E_{m}^{2}G_{m} - \sum_{j=1, j\neq m}^{m} C_{m} \sin \delta_{mj} + D_{mj} \cos \delta_{mj} \right)$$

$$\ddot{q}_{1} + \beta_{1} \left( \sin(q_{1} + q_{10}) - \sin q_{10} \right) = 0$$

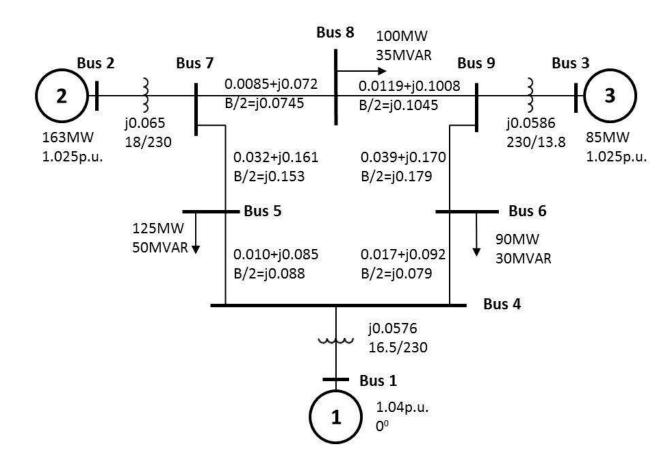
$$\ddot{q}_{2} + \beta_{2} \left( \sin(q_{2} + q_{20}) - \sin q_{20} \right) = 0$$

$$\ddot{q}_{m-1} + \beta_{m-1} \left( \sin(q_{m-1} + q_{m-1,0}) - \sin q_{m-1,0} \right) = 0$$



# **Case Studies**

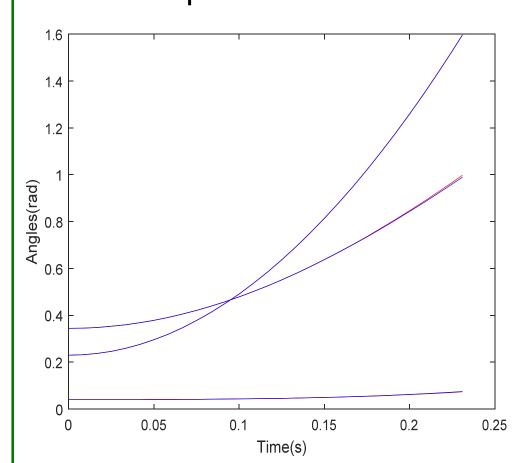
#### **IEEE 3-machine 9-bus system**



# Graph of Time Domain vs. Proposed Method

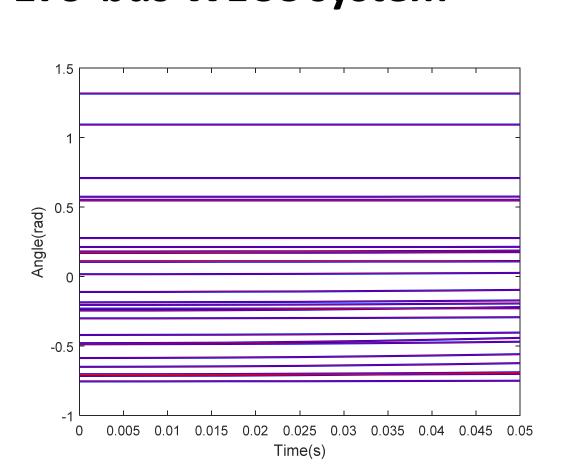
**Accuracy Table** 

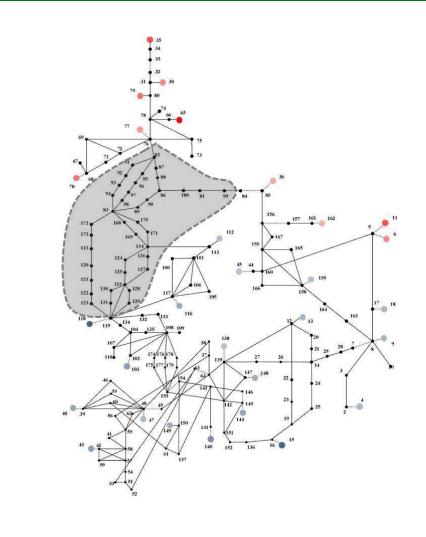
Error Index in Degrees | Error Index in Degrees



CCI	Entor much in Degrees	Elloi maca in Degrees		
	(Fault Duration < CCT)	(Fault Duration = CCT)		
.179	0.001203	0.206169		
.195	0.001203	0.344572		
.231	0.000486	0.491853		
.249	0.000486	0.771834		
.297	0.001325	4.352642		
.324	0.001325	7.209171		
.329	0.000270	1.384967		
.353	0.000721	6.686693		
.430	0.000474	19.936106		
.493	0.000474	41.573108		

## 179-bus WECC system





 20 critical line-tripping contingencies are ranked by DM-II.

Faulted Line	Fault Near	Ranking by ΔV <sub>n</sub>		Ranking	CCT
	Bus		$\Delta V_n$	by CCT	/s
31-80	80	1	-1.000	3	.0431
24-25	24	2	-0.978	1	.0264
22-23	23	3	-0.789	2	.0348
114-171	171	4	-0.417	4	.0489
115-127	127	5	-0.012	5	.0708
130-131	130	6	1.805	6	.0954
108-133	108	7	5.714	15	.3915
14-21	21	8	5.918	10	.2248
19-25	19	9	14.151	11	.2594
83-172	172	10	15.980	7	.1024
104-135	104	11	16.350	17	.5112
48-55	55	12	16.936	18	.5838
136-152	136	13	19.609	16	.4282
41-58	41	14	29.295	19	.7574
49-64	49	15	33.302	20	1.2278
69-72	69	16	33.541	8	.1376
82-87	82	17	69.042	9	.1784
115-127	115	18	129.190	12	.2857
111-173	173	19	176.947	14	.3756
82-91	91	20	247.318	13	.3154

# References

[1] B. Wang, K. Sun and Xiaowen Su, "A decoupling based direct method for power system transient stability analysis," *2015 IEEE PESGM*, Denver, CO, 2015, pp. 1-5.



