Time Series Based Semi-Analytical Solution of Power Systems and its Application in Direct Methods

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Abstract—**This paper proposes a semi-analytical solution that can replace the time domain fault-on trajectory simulation in an adopted decoupling based direct method. All elements of the decoupling based direct method are used except for the fault on trajectory which is calculated using time domain simulation. Instead, a semi-analytical solution is generated based on the transient energy function of the power system. Unknown coefficients are obtained by solving a set of linear equations. These unknown coefficients are then solved for and substituted in order to determine all the values of each of the equations per machine inside of the power system. This method solves these differential equations, where the solution is assumed to follow a certain explicit polynomial in time with unknown coefficients. Once solved, all of the coefficients have a value and make it possible to determine the system trajectory following a certain disturbance under test.**

Index Terms—**semi-analytical solution, direct method, linear decoupling, transient stability, stability margin, transient energy function, decoupled systems, system trajectory.**

I. Introduction

Power systems must work under stable conditions in order to be able function securely. Whenever a fault occurs, the system's stability begins to fluctuate. If the stability reaches a point of no return, then the system completely loses its stability and causes a blackout in an area. In order to take action and prevent the system from losing its stability, there are two different methods that determine whether or not the system is going to remain stable.

The first method is known as the time domain method. This method focuses on creating a numerical solution of the nonlinear differential algebraic equations. Although it is extremely accurate, it does not provide any information on the stability margin of the system and is time consuming. Where there might be a screening of the system done for 30 seconds, the time domain simulation might take around one minute or more to cover those 30 seconds. This means that relying on the time domain simulation to determine if a system will be stable in the near future is not a reliable method. [1]

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The second method is known as the direct method. This method focuses on evaluating a predefined energy function for a power system and works almost in real time. This allows for a screening of the stability of a power system to be quick and somewhat accurate. Although it is not as accurate as the time domain method, it does determine a stability margin with a low error index and can be quite reliable in many cases. There is one issue with the direct method that makes it almost real-time and it is because it uses time domain simulation to determine the fault-on trajectory of the system. [1]

The concept behind a traditional direct method is to determine the kinetic energy of the disturbance on the system and subtract it from the potential energy of the system. The potential energy of the system which is also known as its boundary point is a point that the system cannot pass beyond or else it will completely lose its stability and never return to a stable system again. Once the kinetic energy is subtracted from the potential energy of the system, then the stability margin is determined and if it is positive then the system will remain stable.

There are different ways to incorporate the direct method, and the one that is adopted in this research is known as a decoupling based direct method. This method uses linear decoupling transformation and constructs oscillation modes for single machine infinite bus (SMIB) power systems. The transient energy function method is applied to each of the SMIB systems and then the stability margins are calculated. This allows to deal with each machine as an individual before applying it as a whole to the power system and still accurately calculates the stability margin. [2]

This method follows the steps of a traditional direct method except for the way that it solves for the stability margin. Since it follows the same steps, then that means that it also uses time domain simulation in order to determine the fault-on trajectory of the system. This proposed method uses all of the other steps in this decoupling method except for the time domain simulation that determines that fault-on trajectory. Instead, the proposed method replaces the time domain simulation with a fault-on symbolic semi-analytical solution which can be ran offline and as a result make the method run even faster than real-time. Fast contingency screening techniques are one of the most important parts of dealing with faults/disturbances inside of a power system. This method aims at helping to increase the speed without reducing the accuracy of the decoupling based method.

II. Derivation of Semi-Analytical Solution

A. Semi-analytical solution steps

(1)

$$\dot{x} = f(x)$$

(2)

$$x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{pmatrix} \approx \begin{pmatrix} a_{10} + a_{11}t + a_{12}t^2 + \cdots + a_{1m}t^m \\ a_{20} + a_{21}t + a_{22}t^2 + \cdots + a_{2m}t^m \\ \vdots \\ a_{n0} + a_{n1}t + a_{n2}t^2 + \cdots + a_{nm}t^m \end{pmatrix}$$

Where a_{10} through a_{nm} are unknown variables. Total number of unknowns are (m+1)*n.
Substitute assumed solution (1) into given differential equations (2).

(3)

$$\begin{cases} \dot{x} = f(x) \\ \vdots \\ n \end{cases}$$

Equate the two sides of differential equations term by term for each equation. Since the equations contain a variable ‘t’ that has an order of 1. Then the special condition is met and the equations can be equated in order to find the value of each coefficient behind that ‘t’.

(4)

$$t^0, t^1, \cdots, t^{m-1}, t^m$$

Continuous substitution back into the equations with the new found coefficients would allow for a full solution of each equation until only known coefficients are left. This full solution of coefficients is known as that semi-analytical solution.

In sum, this semi-analytical solution can be created for any system using any number of machines. The only thing that will change in a system with more machines is the amount of equations and coefficients. However, the constant values in the transient energy function will remain the same and will allow for a solution every time.

B. Semi-analytical solution application

After calculating the semi-analytical solution, the next step would be to apply it to the direct method. Since the semi-analytical solution gives us the fault-on trajectory of the system, the last point can be used for calculating what the stability margin is going to be for the system following a disturbance. Once this last point of the fault-on trajectory is discovered, the rest of the method would be to use the decoupling based direct method to analyze the stability of that system.

Therefore, the solution that was derived using the methods in section A, can be directly integrated and completely replace the time domain simulation inside of the decoupling based direct method. This replacement causes the decoupling method which was already close to real time, increase in speed and be able to function faster than it did originally.

III. Case Studies

A. Tests on IEEE 9-bus power system

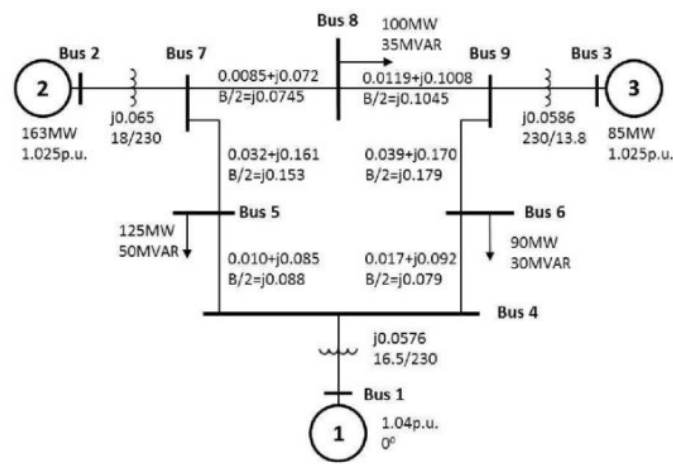

Figure 1. IEEE 9-bus power system

The first test was conducted on the IEEE 3-machine 9-bus power system found in figure 1. The fault-on trajectory was calculated using the proposed method. In order to determine if the proposed method was accurate, it was compared to the time domain simulation which is the accurate fault-on trajectory of the system as seen in the figure below.

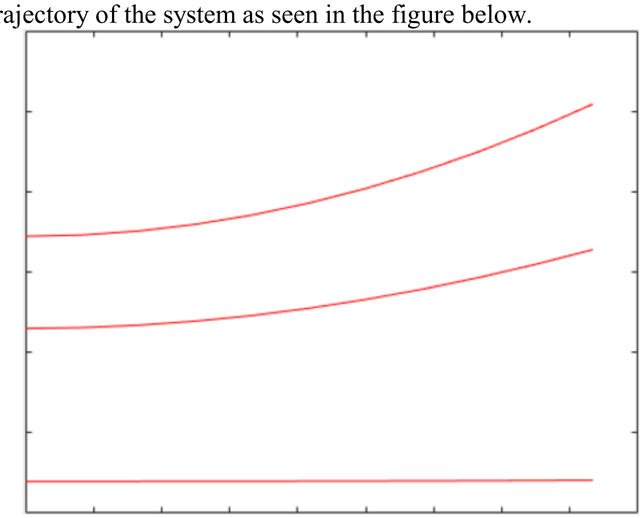

Figure 2. Fault-on trajectory comparison

Figure 2 demonstrates the accuracy of the proposed method. The red curve that is visible is the fault-on trajectory of the proposed method and the blue curve (which cannot be seen under the red curve) is the fault-on trajectory as a result of the time domain simulation portion of the decoupled direct method. Since the red and blue curves are right on top of one another, then the proposed method was able to demonstrate that it could calculate the fault-on trajectory of the system accurately enough to replace the time domain simulation.

Once replaced, the decoupled direct method was used to determine the stability margin of the system for different cases in which a disturbance would affect a certain part of the system.

Faulted Line	Fault Near Bus	Mode 1	Mode 2	Ranking by ΔV_n	ΔV_n	Ranking by CCT	CCT /s
5-7	7	2.090	4.043e+03	1	2.090	1	.179
7-8	7	5.808	169.911	2	5.808	2	.195
5-7	5	8.724	4.499e+03	3	8.724	8	.353
6-9	9	8.991	145.518	4	8.991	3	.231
7-8	8	13.023	80.503	5	13.023	5	.297
8-9	8	20.550	67.292	6	20.550	6	.324
8-9	9	34.971	21.112	7	21.112	4	.249
6-9	6	25.904	1.165e+03	8	25.904	9	.430
4-6	4	33.188	7.355e+03	9	33.188	7	.329
4-6	6	50.981	998.218	10	50.981	10	.493

Figure 3. Stability margin and CCT ranking table

The fault duration in each case was set to be 5/60s which is equal to .083 seconds. As seen in figure 3, there is a small relation between the CCT and the stability margin of each case. However, although the data follows a pattern, determining whether or not it was accurate was the next step.

First, the error index in degrees was calculated for cases in which the fault duration was smaller than the CCT. Then the error index was calculated when the fault duration was equal to the CCT. The reason for testing both less than and equal to was to determine in which case was the proposed method most accurate.

CCT	Error Index in Degrees (Fault Duration < CCT)	Error Index in Degrees (Fault Duration = CCT)
.179	0.001203	0.206169
.195	0.001203	0.344572
.231	0.000486	0.491853
.249	0.000486	0.771834
.297	0.001325	4.352642
.324	0.001325	7.209171
.329	0.000270	1.384967
.353	0.000721	6.686693
.430	0.000474	19.936106
.493	0.000474	41.573108

Figure 4. Stability margin error index

In cases where the fault duration was lower than the CCT, it can be seen in figure 4 that the CCT was directly related with the degree of error in each case. Whenever the fault duration was lower than the CCT, the accuracy went higher and higher per case. However, when they were equal, the accuracy began to decrease as the CCT got larger.

In order to understand why this happens, both cases with the CCT of .231 and .430 were graphed using different fault durations. The case with the CCT of .231 was graphed using a fault time duration that was lower than the CCT in that case. The other case with a CCT of .430 was graphed using a fault duration time that was equal to the CCT as can be seen below.

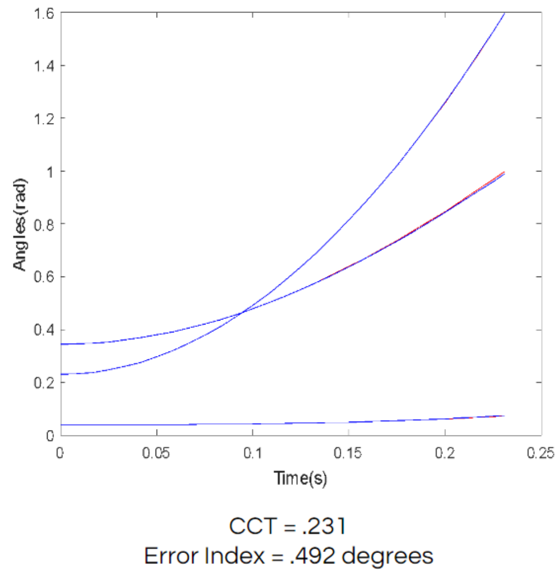

Figure 5. Fault duration time lower than CCT

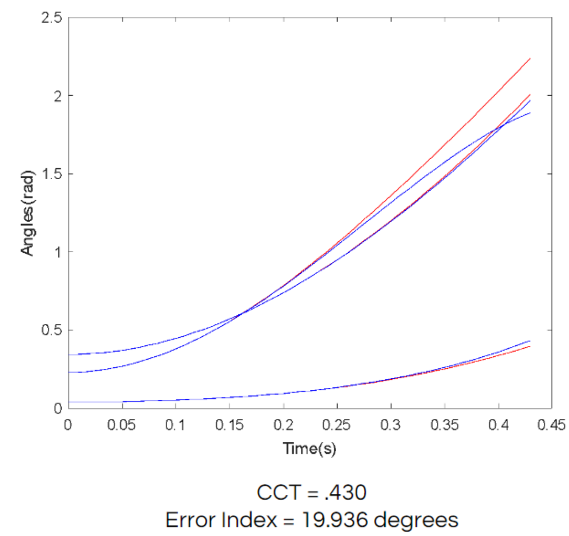

Figure 6. Fault duration time higher than CCT

Both figure 5 and figure 6 demonstrate that the stability margin has a higher accuracy when the simulation does not last long periods of time or when the CCT is equal to the fault duration. However, for shorter periods of time or whenever the CCT is greater than the fault duration, then it does follow the curve and remains accurate. This is useful because it shows that the proposed method can solve for the cases that have a small period of reaction to the fault. By being accurate on smaller faults, this means that the proposed method can be specifically used for determining the stability of a system following an extremely small disturbance which are usually the harder cases to solve for.

B. Tests on WECC 179-bus power system

The second test was conducted on a 29-machine 179-bus model of the WECC power system found in figure 7 below. Since the proposed method proved to be accurate for a smaller system, then it is assumed that it would remain accurate for a larger system.

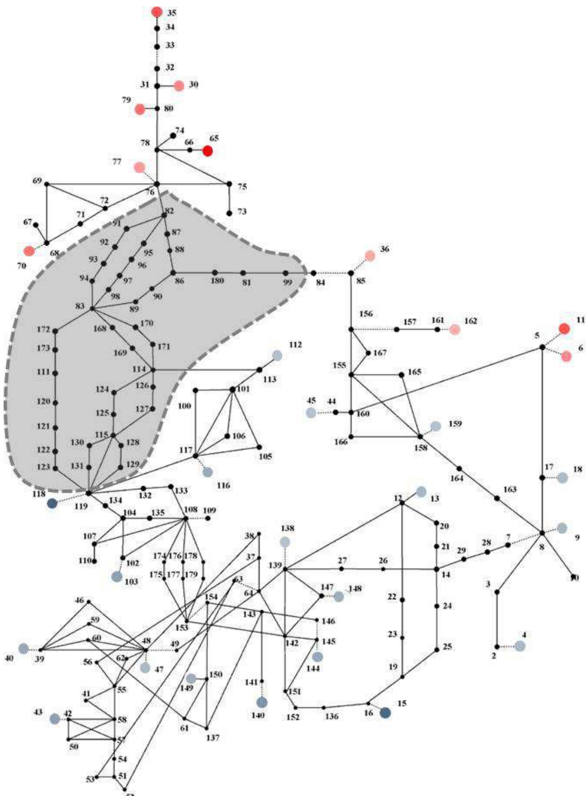

Figure 7. WECC 179-bus power system

Once the stability margins were calculated for each case, they were ranked into a table from lowest stability margin to the greatest. The different cases used in this study were picked every few cases from a low CCT to a higher CCT. Since the fault duration used in each of these cases was .05 seconds, then any CCT that would be lower than .05, would make the system unstable. Any of the cases in which the CCT is greater than .05 should be stable.

In the figure below, it can be seen what the rankings of the stability margins are.

Faulted Line	Fault Near Bus	Ranking by ΔV_n	ΔV_n	Ranking by CCT	CCT /s
31-80	80	1	-1.000	3	.0431
24-25	24	2	-0.978	1	.0264
22-23	23	3	-0.789	2	.0348
114-171	171	4	-0.417	4	.0489
115-127	127	5	-0.012	5	.0708
130-131	130	6	1.805	6	.0954
108-133	108	7	5.714	15	.3915
14-21	21	8	5.918	10	.2248
19-25	19	9	14.151	11	.2594
83-172	172	10	15.980	7	.1024
104-135	104	11	16.350	17	.5112
48-55	55	12	16.936	18	.5838
136-152	136	13	19.609	16	.4282
41-58	41	14	29.295	19	.7574
49-64	49	15	33.302	20	1.2278
69-72	69	16	33.541	8	.1376
82-87	82	17	69.042	9	.1784
115-127	115	18	129.190	12	.2857
111-173	173	19	176.947	14	.3756
82-91	91	20	247.318	13	.3154

Figure 8. Large system stability and CCT ranking table

In Figure 8, all the cases followed the rule that if the CCT was lower than .05 then the system would unstable except for one case out of them all. The case that wars ranked fifth in terms of stability margin was the only one that did not fit that rule. The main reason being that the proposed method is conservative in this regard. Since the stability margin was so close to 0, it lables it as negative in order to make sure that the case is checked over in order to prevent the system from losing its stability.

IV. Conclusions

This paper demonstrates the derivation of the semi-analytical solution that can be used in a decoupled direct method in order to speed up run time. The proposed method does not make the solution inaccurate since the degrees of error for a short fault time duration has low degrees of error. The proposed method also works for both small systems as well as large systems. After all, it is a general solution to a power system with any number of machines.

V. Acknowledgements

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VI. References

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